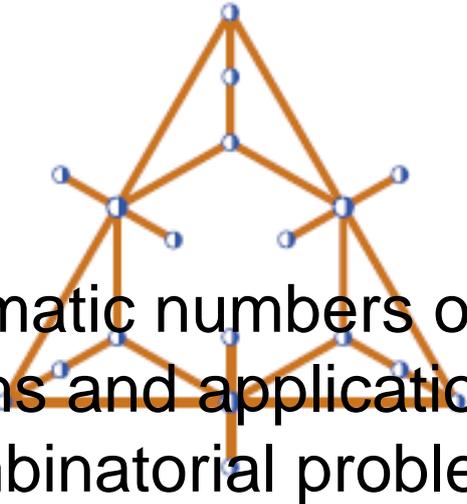


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The chromatic numbers of distance graphs and applications to combinatorial problems.

Content :

We consider distance graphs in \mathbb{R}^n . The sets of vertices of these graphs are subsets of \mathbb{R}^n . Two vertices are connected by an edge if the distance between them is exactly d . This work deals with distance graphs $G = G(n, m, \{a_0, a_1, \dots, a_m\}, x) = (V, E)$ of certain type. Namely, such graphs have the sets of vertices

$$V = \{y = (y_1, \dots, y_n), y_i \in \{0, 1, \dots, m\}, \\ \forall i: y_i = j \Rightarrow a_j = a_{j+1} \text{ for all } j = 0, \dots, m, a_0 + a_1 + \dots + a_m = 1\}$$

and the sets of edges $E = \{(y_1, y_2) \mid y_1, y_2 \in V, (y_1, y_2) = xn\}$.

We managed to prove the following theorem.

Let $m=1, a_1=a, x>0, a, x \in \mathbb{Q}$, and consider the sequence of dimensions n_1, n_2, \dots which satisfies the condition $a_{n_i}, x_{n_i} \in \mathbb{N}$. Consider distance graphs $G_i = G(n_i, 1, \{1-a, a\}, x)$. Let k be a natural number, $k \geq 3$.

$$x < \frac{(ka)^2 - (ka)^2 - [ka]}{k(k-1)} = f_1,$$

then G_i do not contain complete subgraphs (cliques) on k vertices.

Moreover, this bound is in some sense sharp. Namely, there exists a constant $c=c(k, a)$, such that in the sequence of dimensions n_1, n_2, \dots graphs $\tilde{G}_i = G(n_i, 1, \{1-a, a\}, f_1)$ contain complete subgraphs on k vertices.

We proved similar theorems for bigger values of m . We applied these results to the following problem. Let G be a distance graph in \mathbb{R}^n , that does not contain cliques of size k . How large can the chromatic number of such graph be?

This problem is related to the classical problem of finding the value $\chi(\mathbb{R}^n)$ which is equal to the minimum number of colors needed to paint all the points in \mathbb{R}^n so that any two points at distance 1 apart receive different colors. It is known, that $(1.239 + o(1))^n \leq \chi(\mathbb{R}^n) \leq (3 + o(1))^n$.

We obtained new exponential lower bounds for the chromatic number of k -clique-free distance graphs for small $k \geq 4$.

Another application is related to well-known Borsuk's problem -- the problem of finding the minimum number $f(n)$ of parts of smaller diameter, into which an arbitrary set of diameter 1 in \mathbb{R}^n can be divided. Famous Borsuk's conjecture states that $f(n) = n+1$? We study some modification of Borsuk's problem. Namely, we consider sets of diameter 1, lying on the spheres S_{r}^{n-1} of radius r . Using graphs of the above described type, we managed to prove the following theorem:

Theorem 1. Let $S_{r}^{n-1} \subset \mathbb{R}^n$ be the sphere of radius r with center at the origin.

For any $r > \frac{1}{2}$, there exists a $n_0 = n_0(r)$ such that for every $n \geq n_0$,

one can find a set $\Omega \subset S_{r}^{n-1}$ which has diameter 1 and does not admit a partition into $n+1$ parts of smaller diameter.

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