Bled'11 - 7th Slovenian International Conference on Graph Theory

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Tridiagonal pairs and distance-regular graphs

Content :

Let $\mathbb F$ denote a field and let V denote a vector space over $\mathbb F$ with finite positive dimension. We consider a pair of linear transformations $A:V \to V$ and $A^*:V \to V$ that satisfy the following conditions:
\begin{enumerate}
\item[\rm (i)] Each of A,A^* is diagonalizable.
\item[\rm (ii)] There exists an ordering $\lbrace V_i\rbrace_{i=0}^d$ of the eigenspaces of A such that
\begin{eqnarray*}
A^* V_i \subseteq V_{i-1} + V_{i} + V_{i+1} \qquad (0 \leq i \leq d),
\end{eqnarray*}
where $V_{-1}=0$ and $V_{d+1}=0$.
\item[\rm (iii)] There exists an ordering $\lbrace V^*_i\rbrace_{i=0}^\delta$ of the eigenspaces of A^* such that
\begin{eqnarray*}
A V^*_i \subseteq V^*_{i-1} + V^*_{i} + V^*_{i+1} \qquad (0 \leq i \leq \delta),
\end{eqnarray*}
where $V^*_{-1}=0$ and $V^*_{\delta+1}=0$.
\item[\rm (iv)] There is no subspace W of V such that $AW \subseteq W$, $A^* W \subseteq W$, $W \neq 0$, $W \neq V$.
\end{enumerate}
We call such a pair a {\it tridiagonal pair} on V.

In the first part of the talk we classify up to isomorphism the tridiagonal pairs over an algebraically closed field.
In the second part of the talk we discuss how tridiagonal pairs arise in algebraic graph theory. The connection is summarized as follows. For each tridiagonal pair the members of the pair satisfy two cubic polynomial relations called the {\it tridiagonal relations}. The corresponding {\it tridiagonal algebra} T is defined by two generators subject to those relations. The algebra T is noncommutative and infinite-dimensional. Let Γ denote a Q-polynomial distance-regular graph with vertex set X. Fix $x \in X$. Then there exists a tridiagonal algebra T over $\mathbb C$ and a representation $\rho:T\to {\rm Mat}_X(\mathbb C)$ such that both
(i) $\rho(A)$ is the $(0,1)$-adjacency matrix of Γ;
(ii) $\rho(A^*)$ is diagonal with (y,y)-entry

θ^*_i, where i denotes the path-length distance between x,y and
θ^*_i is the ith dual eigenvalue of the Q-polynomial structure. The image
$\rho(T)$ coincides with the subconstituent algebra of Γ with respect to x.
This is joint work with Tatsuro Ito and Kazumasa Nomura.

\begin{thebibliography}{10}

\bibitem{TersubI}
P.~Terwilliger.
\newblock The subconstituent algebra of an association scheme I.
\newblock {\em J. Algebraic Combin.} {\bf 1} (1992) 363--388.

\bibitem{TD00}
T.~Ito, K.~Tanabe, and P.~Terwilliger.
\newblock Some algebra related to ${P}$- and ${Q}$-polynomial association schemes,
in:
\newblock {\em Codes and Association Schemes (Piscataway NJ, 1999)}, Amer. Math.
Soc., Providence RI, 2001, pp. 167--192; {\tt arXiv:math.CO/0406556}.

\bibitem{class}
T.~Ito, K.~Nomura, P.~Terwilliger.
\newblock A classification of the sharp tridiagonal pairs.
\newblock{\em Linear Algebra Appl.} Submitted 2010; {\tt arXiv:1001.1812}.

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