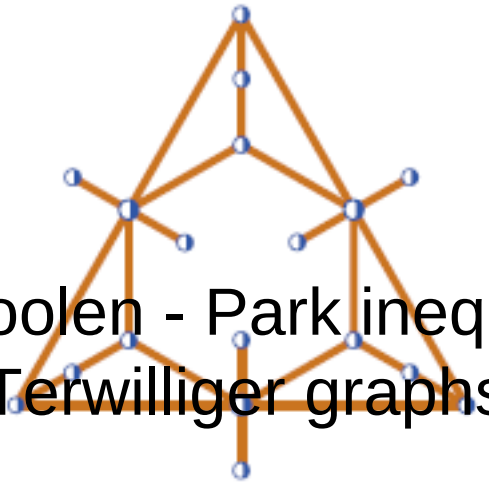
Bled'11 - 7th Slovenian International Conference on Graph Theory



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On the Koolen - Park inequality and Terwilliger graphs

Content :

A Terwilliger graph is a connected non-complete graph Γ such that, for any two vertices $u,w\in \Gamma$ at distance 2, the subgraph induced by the common neighbors of u and w is a complete graph of size μ (for some fixed $\mu \ge 1$). There are only three distance-regular Terwilliger graphs known with $\mu\ge 2$, all of them are characterized by theirs intersection arrays. The three examples are:

(1) the icosahedron with intersection array $\{5, 2, 1; 1, 2, 5\}$ is locally pentagon graph;

(2) the Doro graph with intersection array $\{10, 6, 4; 1, 2, 5\}$ is locally Petersen graph;

(3) the Conway–Smith graph with intersection array $\{10, 6, 4, 1; 1, 2, 6, 10\}$ is locally Petersen graph.

Let Γ be a distance-regular graph with intersection array $\{b_0,b_1,...,b_{d-1};c_1,c_2,..,c_{d}\}$ and diameter $d\ge 2$. Let c be maximal such that, for each vertex $x\in \Gamma$ and every pair of nonadjacent vertices y,z of $\Gamma_1(x)$, there exists a c-coclique in $\Gamma_1(x)$ containing y,z. J.H. Koolen and J. Park showed that the following bound holds:

$$c_2-1\ge \frac{c(b_0-b_1)-b_0}{{c \choose 2}},$$

and equality implies that Γ is a Terwilliger graph.

We prove that if the Koolen - Park bound is attained, then $c_2=2$ and Γ is the icosahedron, the Doro graph or the Conway–Smith graph.

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