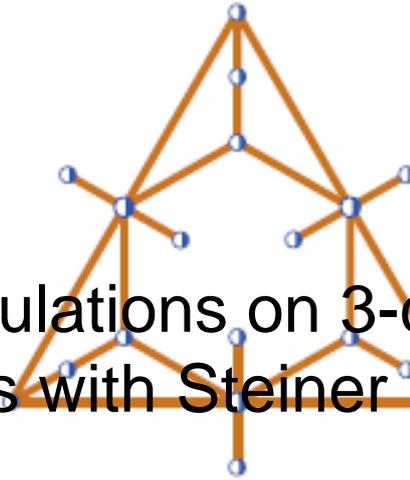


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## Quadrangulations on 3-colored point sets with Steiner points



### Content :

Let  $P$  be a point set on the plane, and consider whether  $P$  is  $\{k$ -quadrangulatable $\}$ , that is, whether there exists a 2-connected bipartite plane graph  $G$  with edge edge straight segment such that  $V(G)=P$ , that the outer cycle of  $G$  coincides with  $\text{Conv}(P)$ , that each finite face of  $G$  is quadrilateral. It is easy to see that it is possible if an even number of points of  $P$  lie on  $\text{Conv}(P)$ . Hence we give a  $k$ -coloring of  $P$  and consider the same problem, avoiding edges joining two vertices of  $P$  with the same color. In this case, we always assume that the number of the points of  $P$  lying on  $\text{Conv}(P)$  is even and any two consecutive points on  $\text{Conv}(P)$  have distinct colors. However, there is a 2-colored non-quadrangulatable point set  $P$ . So we introduce a  $\{k$ -Steiner point $\}$ , which can be put in any position of the interior of  $\text{Conv}(P)$  and may be colored by any color of the  $k$  colors. When  $k=2$ , Alvarez et al. proved that if a point set  $P$  on the plane consists of  $\lfloor \frac{n}{2} \rfloor$  red and  $\lfloor \frac{n}{2} \rfloor$  blue points in general position, then adding Steiner points  $Q$  with  $|Q| \leq \lfloor \frac{n-2}{6} \rfloor + \lfloor \frac{n+4}{6} \rfloor + 1$ ,  $P \cup Q$  is quadrangulatable, but there exists a non-quadrangulatable 3-colored point set for which no matter how many Steiner points are added. In this paper, we define the winding number for a 3-colored point set  $P$ , and prove that a 3-colored point set  $P$  in general position with a finite number of Steiner points  $Q$  added is quadrangulatable if and only if the winding number of  $P$  is zero. When  $P \cup Q$  is quadrangulatable, we prove  $|Q| \leq \frac{7n+34m-48}{18}$ , where  $|P|=n$  and the number of points of  $P$  in  $\text{Conv}(P)$  is  $2m$ .

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