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On the chromatic number of q-Kneser graphs

Content :

The q-analogue of questions about sets and subsets are questions about vector spaces and subspaces. For a prime power q, and an n-dimensional vector space V over $GF(q)$, let ${V\brack k}$ denote the family of k-subspaces of V.

The vertex set of the q-Kneser graph $qK_{n:k}$ is ${V\brack k}$, where V is an n-dimensional vector space over $GF(q)$. Two vertices of $qK_{n:k}$ are adjacent if and only if the corresponding k-subspaces are disjoint (i.e., meet in 0).

We first prove the q-analogue of the theorem of Hilton-Milner \cite{HM}, then apply it to determine the chromatic number of q-Kneser graphs.

Theorem (\cite{BBCFPMS}, \cite{CGR} for $k=2$).
If $k \geq 3$ and $q\geq 3$, $n \geq 2k+1$ or $q=2$, $n \geq 2k+2$, then for the chromatic number of the q-Kneser graph we have $\chi(qK_{n:k}) = {n-k+1\brack 1}$. Moreover, each colour class of a minimum colouring is a point-pencil and the points determining a color are the points of an $(n-k+1)$-dimensional subspace.}

The case $n=2k$ is more interesting (and seems to be more difficult). In this case Godsil and Newman \cite{GN} characterized the largest cocliques and we could show that the second largest coclique must be essentially smaller if $q>q_k$ is large enough.
We conjecture that the chromatic number of $qK_{2k:k}$ equals q^k+q^{k-1}, for all q and k, unfortunately we were only be able to prove this when q is large enough compared to k, see \cite{BBS}. The case $n=4$, $k=2$ was done earlier by Eisfeld, Storme, Sziklai \cite{ESS01} and Chowdhury, Godsil, and Royle \cite{CGR}.
Fpr $k=3,n=6$ we could completely describe the cocliques that are larger than $3q^4+3q^3+2q^2+q$, see \cite{BBS}.

This is joint work with Aart Blokhuis, Andries Brouwer and other colleagues.

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Primary authors : Prof. SZONYI, Tamas (Eotvos Lorand University and Computer and Automation Institute HAS)

Co-authors :

Presenter : Prof. SZONYI, Tamas (Eotvos Lorand University and Computer and Automation Institute HAS)

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