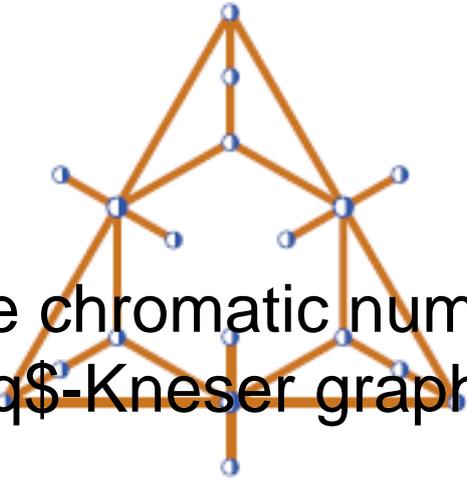


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On the chromatic number of q -Kneser graphs

Content :

The q -analogue of questions about sets and subsets are questions about vector spaces and subspaces. For a prime power q , and an n -dimensional vector space V over $\text{GF}(q)$, let $\{\mathcal{V} \backslash \text{brack } k\}$ denote the family of k -subspaces of V .

The vertex set of the q -Kneser graph $qK_{\{n:k\}}$ is $\{\mathcal{V} \backslash \text{brack } k\}$, where V is an n -dimensional vector space over $\text{GF}(q)$. Two vertices of $qK_{\{n:k\}}$ are adjacent if and only if the corresponding k -subspaces are disjoint (i.e., meet in 0).

We first prove the q -analogue of the theorem of Hilton-Milner [\cite{HM}](#), then apply it to determine the chromatic number of q -Kneser graphs.

Theorem ([\cite{BBCFPMS}](#), [\cite{CGR}](#) for $k=2$).

If $k \geq 3$ and $q \geq 3$, $n \geq 2k+1$ or $q=2$, $n \geq 2k+2$, then for the chromatic number of the q -Kneser graph we have $\chi(qK_{\{n:k\}}) = \lfloor n-k+1 \rfloor$. Moreover, each colour class of a minimum colouring is a point-pencil and the points determining a color are the points of an $(n-k+1)$ -dimensional subspace.

The case $n=2k$ is more interesting (and seems to be more difficult). In this case [Godsil and Newman \cite{GN}](#) characterized the largest cocliques and we could show that the second largest coclique must be essentially smaller if $q > q_k$ is large enough.

We conjecture that the chromatic number of $qK_{\{2k:k\}}$ equals $q^k + q^{k-1}$, for all q and k , unfortunately we were only able to prove this when q is large enough compared to k , see [\cite{BBS}](#). The case $n=4$, $k=2$ was done earlier by [Eisfeld, Storme, Sziklai \cite{ESS01}](#) and [Chowdhury, Godsil, and Royle \cite{CGR}](#). For $k=3, n=6$ we could completely describe the cocliques that are larger than $3q^4 + 3q^3 + 2q^2 + q$, see [\cite{BBS}](#).

This is joint work with Aart Blokhuis, Andries Brouwer and other colleagues.

Bibliography:

\bibitem{BBS}

\textsc{A.-Blokhuys, A.-E.-Brouwer, T.-Sz\H{o}nyi},
{em On the chromatic number of q -Kneser graphs},
Designs, Codes, and Cryptography, to appear

\bibitem{BBCFPMS}

\textsc{A.-Blokhuys, A.-E.-Brouwer, A.-Chowdhury, P.-Frankl, B.-Patk\os, T.-Mussche},
T.-Sz\H{o}nyi},
{em A Hilton-Milner theorem for vector spaces},
Electr. J. of Combinatorics {\bf 17} (2010) R71.

\bibitem{CGR}

\textsc{A.-Chowdhury, C.-Godsil, G.-Royle},
{it Colouring lines in a projective space},
J. Combin. Theory Ser. A {\bf 113} (2006) 228--236.

\bibitem{ESS01}

\textsc{J. Eisfeld, L. Storme, P. Sziklai},
{it Minimal covers of the Klein quadric},
J. Combin. Theory Ser. A {\bf 95} (2001) 145--157.

\bibitem{FW}

\textsc{P.-Frankl, R.-M.-Wilson},
{it The Erd\H{o}s-Ko-Rado theorem for vector spaces},
J. Combin. Theory Ser. A {\bf 43} (1986) 228--236.

\bibitem{GN}

\textsc{C. D.-Godsil, M. W.-Newman},
{it Independent sets in association schemes},
Combinatorica {\bf 26} (2006) 431--443.

\bibitem{HM}

\textsc{A. J. W.-Hilton, E. C.-Milner},
{it Some intersection theorems for systems of finite sets},
Quart. J. Math. Oxford Ser. (2) {\bf 18} (1967) 369--384.

\bibitem{H}

\textsc{W. N.-Hsieh},
{it Intersection theorems for systems of finite vector spaces},
Discrete Math. {\bf 12} (1975) 1--16.

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