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Invited Talks

Beyond the Tensor–product Model: Hierarchical Spline Structures

Carlotta Giannelli

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The standard technology of Computer Aided Design relies on Non Uniform Rational B-splines (NURBS) as typical design framework. Properties like local support and non-negativity of basis functions, together with efficient computations through related application algorithms, are the key ingredients which motivate this choice. In addition, thanks to the convex hull property, the user can effectively model and manipulate NURBS representations by simply acting on a corresponding set of control points.

Unfortunately, the regular tensor–product structure behind the NURBS model does not provide local refinement possibilities. For this reason, classical B–spline constructions preclude strictly localized surface modeling or adaptive mesh refinements in the context of isogeometric analysis. The hierarchical approach overcomes this lack of flexibility by considering a multilevel scheme which reflects different levels of resolution.

The talk will review recent results concerning the theoretical characterization and practical use of multilevel B–spline spaces of arbitrary degree. Related issues connected to an increasing need of extending current geometric standards in the framework of up–to–date and application–oriented settings will also be discussed.

Generalized Barycentric Coordinates

Kai Hormann

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Switzerland*

In 1827 August Ferdinand Möbius published his seminal work “Der barycentrische Calcul” which provided a novel approach to analytic geometry. One key element in his work is the idea of barycentric coordinates which allow us to write any point inside a triangle as a unique convex combination of the triangle’s vertices.

Only recently, this idea has been extended to polygons with more than three vertices and arbitrary polytopes in higher dimensions, yielding an efficient method for interpolating data given at the vertices of an arbitrary polytope.

We discuss the theoretical background of these generalized barycentric coordinates and present several useful applications, e.g. in computer graphics, computer aided geometric design, and image processing.

Subdivision Curves and Surfaces: Theory and Applications

Jiří Kosinka

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Cambridge, United Kingdom*

Subdivision is a powerful and popular technique for generating free-form curves and surfaces with many applications in geometric modelling, multiresolution analysis, computer games, and the film industry. Since its introduction to computer graphics in the 1970s it has been developed into a mature technology that is able to compete with other geometry representations, including NURBS.

On the other hand, engineering applications, such as computer-aided design and manufacturing, use predominantly NURBS. A recent NURBS-compatible subdivision framework by Thomas Cashman provides a subdivision mechanism that is a true superset of NURBS. Arbitrary degree NURBS can be represented in Cashman's formulation, with the additional feature that the arbitrary topology of subdivision is also supported.

In this talk, after giving a brief historical overview of subdivision techniques, we will focus on recent and ongoing work in the area of subdivision in the Rainbow Group at Cambridge. We will explore modelling of (semi-sharp) creases in arbitrary degree subdivision curves and surfaces (building on Cashman's framework), subdivision surface fitting, and an application of subdivision surfaces in image processing.

Motion Polynomials and the Combinatorics of Linkages

Josef Schicho

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It is well-known that the group of Euclidean displacements is isomorphic to the multiplicative group of dual quaternions with nonzero real norm modulo nonzero real numbers. We extend this to an isomorphism of the group of rationally parametrized motions and the group of certain polynomials with coefficients in the dual quaternions, also called *motion polynomials*. Algebraic properties of these motion polynomials can be exploited to construct closed linkages with rotational or translational joints of high complexity.

Curve Network-based Design and Multi-sided Transfinite Surfaces

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Creating complex free-form objects using general topology surface models is a major task in computer aided geometric design. A natural approach is (i) to define the most important characteristic curves of the object, (ii) merge these into a single 3D curve network and (iii) interpolate the loops of curves by multi-sided transfinite surfaces. Curves may come from several sources, like traditional blueprints, 2D sketches, feature curves on images, or directly through 3D interactive editing. Transfinite surfaces typically interpolate Hermite data; they are defined as the combination of so-called ribbons that constrain prescribed positional and cross-derivative functions along the boundary curves.

In this talk we analyze the “pros and cons” of curve network-based design by comparing this with other surface modeling approaches, such as the application of (i) trimmed surfaces, (ii) centrally split, composite quadrilateral surfaces, or (iii) recursive subdivision. After briefly reviewing previous work, the basic constituents of transfinite surface interpolation will be discussed, including simple algorithms to generate ribbons from the curve network, creating optimized, non-regular polygonal domains, applying various distance-based blending functions, and computing various ribbon parameterizations, that determine how the points of the individual ribbons are blended together.

Our main interest is to present two recent transfinite schemes: the first is a true, multi-sided generalization of the four-sided Coons patch, that combines side interpolants and correction patches, as in the classical Boolean sum scheme. The second formulation is based on curved ribbons, each one of them interpolating three consecutive boundaries.

The flexibility (and some of the difficulties) of curve network-based design will be demonstrated by a few examples, showing curvature maps and analyzing the computational costs of the algorithms.

Contributions

KSpheres – An Efficient Algorithm for Joining Skinning Surfaces

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Interpolation of geometric data sets is of central importance in Computer Aided Geometric Design. Nowadays there is a growing demand to extend the known methods based on points to new methods which rely on other types of geometric objects such as circles and spheres.

Surface modeling based on such principles has already appeared in computer graphics and in the last few years, several scientific papers have been published in this topic. Popular CAD softwares (e.g. ZBrush[®], Spore[™]) seem to apply similar methods and these implementations are available for everyday users. The methods used in these applications, however, do not provide results good enough especially in those cases when we would like to join skinning surfaces to each other.

Joining of given geometric objects is an important part of surface modeling since in this way more complicated objects can be constructed than the starting forms. Therefore there is a persistent demand from both designers and users to use sophisticated methods which provide more freedom throughout the designing process.

We present an efficient algorithm for the joining and branching problem which gives visually much more satisfactory results than the presently available ones. Our method is also presented in our newly developed, user friendly modeling software based on projective geometry.

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C^2 Hermite Interpolation by Pythagorean-hodograph Quintic Triarcs

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In this talk, we propose a new method for solving the problem of C^2 Hermite interpolation in plane and space with the help of triarcs composed of Pythagorean Hodograph (PH) quintics. The main idea is to join three arcs of PH quintics at two unknown points – the first curve interpolates given C^2 Hermite data at one side, the third one interpolates the same type of given data at the other side and the second arc is joined to the first and the third arc with C^2 continuity. For any set of C^2 planar boundary data (two points with the associated first and second derivatives) we construct four possible interpolants. Analogously, for the set of C^2 spatial boundary data we find a four-dimensional family of interpolating PH quintic triarcs. We prove that

the best possible approximation order in both planar and spatial case is 4.
The designed algorithms are presented on several examples.

Approximate Parameterization of Implicit Blends of Canal Surface Type

Michal Bizzarri

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We will present a method for the computation of approximate rational parameterizations of implicitly given canal surfaces. The designed approach is mainly suitable for implicit surfaces of canal surface type which are used for the construction of a smooth transition between two primary shapes. A distinguished feature of our method is a combination of symbolic and numerical techniques yielding approximate topology-based cubic parameterizations of contour curves which are then applied to compute an approximate parameterization of the given canal surface. Some improvements of the traditional approach (e.g. yielding the low rational bidegree of the obtained parameterizations) are presented and discussed. We present several examples which illustrate the functionality of the designed method.

On the Splitting Problem of Orbifolds via D-symbols

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Any tiling, generated by a crystallographic group with compact fundamental domain, can be represented by a diagram and a matrix valued function, based on their barycentric subdivision and the adjacency relations between the orbits and the particular simplices. The representation is called the D-symbol of a tiling in honour of Delone, Delaney and Dress. The representation is easily adoptable to computer programs.

Based on Thurston's geometrization conjecture there are 8 possible geometric structures on special 3-manifolds which are cut along tori. There exists at least 4 proof of the theorem but none of them is constructive. Based on our method it would be easier to find a tiling which does not fit in any of the 8 geometries; but inspecting the tilings we can possibly move forward to a constructive proof of the theorem.

Using D-symbols one can examine the properties of orbifolds, but luckily the 3-manifolds in the conjecture are trivially orbifolds. There are infinitely many D-symbols, but they can be enumerated based on their cardinality. So it may be possible to enumerate every 3-manifold and verify the conjecture.

But first we have to find the "cuts along tori" which are called splittings in our theory. We would like to present an algorithm with some examples for finding every possible splittings of a D-symbol by examining the signature of 2-dimensional subtilings.

The future: After splitting D-symbols along the previously found splittings we have to be able to tell the signature of the underlying projective space of the primitive D-symbols.

Determinantal Representations of Cubic Curves

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For every (irreducible) cubic curve all nonequivalent determinantal representations are explicitly constructed. We can clearly tell which and how many of these representations are self-adjoint. We give a new proof of the criteria when a self-adjoint determinantal representation is definite; in other words, when it is a LMI representation of a spectrahedron on which semidefinite programming can be performed.

Isoptic Curves of Generalized Conic Sections in Hyperbolic Geometry

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In this presentation we consider the isoptic curves on the 2-dimensional geometries of constant curvature $\mathbf{E}^2$, $\mathbf{H}^2$, $\mathcal{E}^2$. The topic is widely investigated in the Euclidean plane $\mathbf{E}^2$, but in the hyperbolic and elliptic plane there are few results in this topic. We show a procedure to study the isoptic curves in the hyperbolic and elliptic plane geometries and apply it for some geometric objects, especially the generalized conic sections in the hyperbolic geometry. We use for the computations the classical models which are based on the projective interpretation of the hyperbolic and elliptic geometry and in this manner the isoptic curves can be visualized on the Euclidean screen of computer.

Support Function Based Description of Topology of Real Algebraic Curves

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The support function representation can considerably simplify each of main phases of the problem of describing the topology of real algebraic curves - the determination of critical points and the reconstruction of their connectivity as well as the consecutive feature-preserving approximation of the topological graph. The connectivity of the topological graph can be determined in a geometrical way without knowledge of the position of self-intersections of the curve. Our algorithm is based on the fact that using support function we can easily find all points with given G^1 data. This allows us to subdivide the maximal possible angle between normals at vertices of the graph and using geometric criteria find the connectivity of the graph. After determination of the connectivity we can describe the position of self-intersections.

Biharmonic Curves in $\widetilde{SL(2, \mathbb{R})}$ Geometry

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A map $\phi : (M, g) \rightarrow (N, h)$ between two Riemannian manifolds is said to be **harmonic** if it is a critical point of the energy functional $E(\phi) := \frac{1}{2} \int_M |d\phi|^2 v_g$. A harmonic map is characterized by the vanishing of the first tension field $\tau(\phi) := \text{trace} \nabla d\phi$.

A map $\phi : (M, g) \rightarrow (N, h)$ is said to be **biharmonic** if it is a critical point of the bienergy functional $E_2(\phi) := \frac{1}{2} \int_M |\tau(\phi)|^2 v_g$. A biharmonic map is characterized by the vanishing of the second tension field $\tau_2(\phi) := J(\tau(\phi))$.

Fifteen years ago Emil Molnár proposed a projective spherical model (as unified geometrical model) of the eight homogeneous Thurston 3-geometries (E^3 , S^3 , H^3 , $S^2 \times \mathbb{R}$, $H^2 \times \mathbb{R}$, $\widetilde{SL(2, \mathbb{R})}$, Nil , Sol) and introduced the hyperboloid model of $\widetilde{SL(2, \mathbb{R})}$ geometry.

Using this hyperboloid model, we examine biharmonic curves in $\widetilde{SL(2, \mathbb{R})}$ geometry. The main result is that only proper biharmonic curves parametrized by arc length are helices.

Segmentation of Planar Computational Domains Using Harmonic Mappings

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Austrian Academy of Sciences, Linz, Austria*

In this talk we show how it is possible to achieve a segmentation of a computational domain into subdomains, employing harmonic mappings and the numerical curve-tracing of the preimage of patches defining a target domain.

Our input is a boundary representation, by means of B-splines or NURBS boundaries, of the computational domain. The first step consists in creating a parameterization of it; this is achieved using an isogeometric boundary element method, where a harmonic mapping is employed in order to map our computational domain to a chosen template.

Once a suitable target domain is chosen from a list of predefined templates, such that it matches the topology of the computational domain, we trace the preimage of the patches, defining the target, inside the computational domain using a predictor-corrector method.

Under two assumptions the existence of a bijective harmonic mapping can be guaranteed. First, the target domain, seen as a collection of tensor-product patches, has to be convex, and second, any interior holes have to be suitably placed.

The traced curves provide us with the segmentation of the domain into quadrilateral domains that are readily fitted with tensor-product Coons patches.

Conchoid Surfaces of Quadrics

David Gruber

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Technology, Vienna, Austria*

The conchoid of a surface F with respect to given fixed point O is roughly speaking the surface obtained by increasing the radius function with respect to the reference point O by a constant d .

We study real quadrics in this context and prove that their conchoids possess real rational parameterizations, independently on the position of the reference point. Thus any real quadric F admits a rational radius function $r(u, v)$ with respect to any point in space. By increasing this radius function to $r(u, v) + d$, the conchoids of F are represented in an analogous way. We present an algorithm to compute the radius function and the resulting real rational parameterization of the quadric and its conchoids. This parameterization is closely related to real rational parameterizations of del Pezzo surfaces of degree four.

Besides the general case we study singular quadrics and present simplifications of the construction in case that $O \in F$ or that O is contained in a focal conic of F .

Blending of Spheres by Rotational-minimizing Surfaces

Miklós Hoffmann

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Juan Monterde

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Computation of the blending surface of two given spheres is discussed in this talk. The blending surface (or skin) has to touch the given spheres at predefined circles. The main advantage of the presented method over the existing ones is the minimization of unwanted distortions of the blending surface. This is achieved by the application of rotational-minimizing frames for the transportation of a vector along a given curve, which technique, beyond its theoretical interest, helps us to determine the corresponding points along the touching circles of the two spheres. Parametric curves of the blending surface are also defined by the help of the rotational minimizing transportation.

Figure 1: Comparison of results of the earlier method (left) and our method (right). Surfaces at the right side have visibly less distorted shape.

Geometric Interpolation by Bivariate Parametric Macro-elements

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Tadej Kanduč

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In this talk, Hermite interpolation by two types of parametric C^1 macro-elements on triangulations is considered. Cubic triangular splines interpolate points and the corresponding tangent planes at domain vertices and approximate tangent planes at midpoints of domain edges. Quintic splines additionally interpolate normal curvature forms at the vertices. Control points of the interpolants are constructed in two steps. In the first one, uniformly distributed control points of a linear spline interpolant are projected to the interpolation planes. To satisfy smoothness conditions between patches, a correction of control points is obtained as a solution of a least square minimization. The construction of the approximant is local and fast. Some numerical examples of surface approximations are presented.

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Geometric Problems inspired by Robotics

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We describe the nature of the basic geometric problem for parallel manipulators. On the basis of this problem we formulate similar questions.

They can be solved on a computer or even by elementary means at high school and undergraduate level. Such problems could increase interest of students in classical geometry and make it more attractive.

For demonstration let us mention the problem of constructing a square with vertices on given four lines.

Canonical Principal Parameters On Minimal Surfaces

Ognian Kassabov

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Ganchev has recently proposed a new approach to minimal surfaces. Introducing canonical principal parameters on any minimal surface, he has proved that the normal curvature determines the surface up to its position in the space. Here we prove a theorem that permits to obtain equations of a minimal surface in canonical principal parameters and we make some applications. Thus we show that the Ganchev's approach implies an effective method to prove the coincidence of two minimal surfaces given in isothermal coordinates by different parametric equations.

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Curvature Approximation of Circular Arcs by Low-degree Parametric Polynomials

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In this talk, approximation of circular arc and its curvature profile using Bézier curves will be presented. Firstly, interpolation by geometrically continuous (G^1) parametric polynomial of order three is considered. Free parameter is chosen in order to minimize curvature error. Necessary and sufficient conditions for the uniqueness of the solution are established. Optimal asymptotic approximation order will also be confirmed. Secondly, interpolation by geometrically continuous (G^1) parametric polynomial biarcs of low degree with at least G^1 joint are considered. Comparison with the well-known methods for minimization of radial error will be shown through numerical examples.

Acknowledgment:

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Skinning of Spheres using Biarcs

Roland Kunkli

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We have developed an algorithm at 2010 with which we can easily construct smooth, tube-like skinning surfaces based on spheres using G^1 continuous Hermite curves. A method which uses only biarc curves for the interpolation is discussed in this presentation. The construction of the touching circles of the curve is based on our earlier results. With this new technique we can give a smoother result in certain situation and can avoid possible intersections between the constructed skinning surface and the spheres.

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Rational Generalized Contour Curves on Canal Surfaces

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We will study a condition which guarantees that a given canal surface has rational generalized contour curves (i.e., contour curves with respect to a given direction), which are later used for a straightforward computation of rational parameterizations of canal surfaces providing rational offsets. Our approach follows a construction of rational spatial MPH curves from the associated planar PH curves and gives it to the relation with the contour curves of canal surfaces given by their medial axis transforms. Special cases (quadratic and cubic MATs) will be thoroughly analyzed and related to the problem of general contour curves. Finally, two simple algorithms for a construction of PN blends between two canal surfaces will be presented.

Minimal Bézier Surfaces Interpolating Two Geodesics Curves

Alfonso Carriazo Rubio

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M. Carmen Marquez Garcia

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Given two cubic curves we consider Bézier patches interpolating these two curves such that they are geodesics of the Bézier surfaces. We study the problem of finding a Bézier surface that minimizes the area from among all these interpolation Bézier patches obtaining conditions to be extremal of the Dirichlet function.

On the Dimension of Hierarchical B-spline Spaces

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Austria*

Local refinement is still a great challenge in Isogeometric Analysis. As an alternative to the T-spline framework, the potential of hierarchical B-splines as trial functions has been recently identified. Apart from practical issues, also a theoretical question about the dimension of the space induced by the hierarchical basis has arisen. The talk will address this question by investigating the relation between the basis and spline spaces in respective levels. Furthermore, reasonable assumptions regarding the domain configuration will be identified.

Visualization of Regular 4-polytopes

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At the Vorau Conference on Geometry in 2007 the presenter with János Katona initiated the computer animation of higher dimensional polytopes especially on the computer screen, by central projection from a complementary centre figure, with visibility effects (see e.g. [1]). All these are presented in the linear algebraic machinery of real projective sphere PS^4 or space $P^4(\mathbf{V}^5, \mathbf{V}_5, \sim)$ over the real vector space $\mathbf{V}^5$ for points and its dual $\mathbf{V}_5$ for hyperplanes up to the usual equivalence $\sim$ (expressed by multiplication by positive real numbers or non-zeros, respectively). All these extend the 3-dimensional constructions of the regular Platonic solids from the homepage <http://www.math.bme.hu/~prok> (for free download) In this presentation we further develop the exterior (Grassmann) algebra method (with scalar product) by computer to other effects of illumination. As new examples, regular 4-polytopes move in the computer 2-screen with visibility and shading of 2-faces, on the base of the above homepage Our illustrations here show two of the six regular 4-solids by their Coxeter - Schläfli symbols on the base of the above homepage, where we further develop this attractive topic. This animations can call the attention to the multilinear algebra which have many applications, e.g. in economics. The use of computer is essential to carry out this research.

References:

- [1] J. Katona – E. Molnár – I. Prok – J. Szirmai: Higher-dimensional central projection into 2-plane with visibility and applications. Kragujevac Journal of Mathematics Volume 35 Number 2 (2011), 249–263.

Figure 2: The 4-cube with Coxeter-Schläfli symbol $(4, 3, 3)$

Figure 3: The 120-cell with Coxeter-Schläfli symbol $(5, 3, 3)$

Method for Optimization of Camera Movement Path Based on Isoptic Curves

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In computer graphics and geometric modeling for good quality displaying of an object it is required that the object must fit on the screen. It often happens, for example when we are using modeling software, that the object we would like to rotate or edit from another point of view, is partly out of the screen, and thus some parts are not visible. The purpose is to find a general method or algorithm that helps to locate the closest possible position for the camera.

In computer modeling the Bézier and B-spline curves and surfaces are standard tools. Therefore the methods that we want to show handle these objects. In two dimensions the isoptic curve of a curve is constructed by involving lines with a given angle intersect each other at a certain point of the isoptic curve. In three dimensions the points of an isoptic curve may be the possible positions of the camera. So from these points we can watch the object with respect to the given viewing angle. The developed algorithm produces an isoptic curve for a special case.

The generalization of the method gives the opportunity to produce isoptic curves in more complicated cases.

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Introducing the Theory of Bonds for Stewart Gough Platforms with Self-motions

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The talk is based on a manuscript of the author, which has the identical title and is currently under review.

Within this presentation, we transfer the basic idea of bonds, introduced by HEGEDÜS, SCHICHO and SCHRÖCKER for overconstrained closed chains with rotational joints, to the theory of self-motions of parallel manipulators of STEWART-GOUGH (SG) type. We give some basic facts and results on bonds and demonstrate the potential of this theory on the basis of several examples, which also show that the bond theory can be used for different tasks, e.g.:

1. Classification of SG platforms with non-translational self-motions.
2. Checking whether given SG platforms are free of non-translational self-motions.
3. Determination of SG platforms with non-translational self-motions.

Moreover, we present a geometric characterization of all SG platforms with a pure translational self-motion.

In addition, we give the results of our recent work on SG manipulators with multidimensional self-motions, which is currently in preparation. The key to a successful study of this topic is hidden in the subdivision of these self-motions into different types, which are induced by the theory of bonds in a natural way. Based on these preparatory considerations, we present a complete list of all SG platforms, which possess n -dimensional self-motions with $n > 2$. We also give some remarks and a new result on SG platforms with 2-dimensional self-motions, nevertheless a full discussion of this case remains open.

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Direct Symbolic Transformation from 3D Cartesian into Hyperboloidal Coordinates

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A direct transformation from Cartesian coordinates into hyperboloidal coordinates (considered for rotational hyperboloids), based on symbolic techniques (mainly Sturm-Habicht sequences and its properties related to real root counting), is presented. The transformation problem is reduced to the problem of finding the positive roots of a fourth degree polynomial. The analysis of the polynomial's roots is performed by a algebraically complete stratification of the plane's positive quadrant. Two approaches for computing the polynomial's roots are presented, one based on the Merriman method and the other one obtained using the Computer Algebra System **Maple**. Our approach improves the solution presented in Feltens *Hyperboloidal coordinates: transformations and applications in special constructions* Journal of Geodesy 2011, being reduced to a few evaluations of symbolic expressions. Hyperboloidal coordinates (λ, ϕ, h) for hyperboloids of one sheet were first introduced in the literature in Feltens *Hyperboloidal coordinates: transformations and applications in special constructions* Journal of Geodesy 2011, together with two iteration processes for the transformation of the 3D Cartesian coordinates (X, Y, Z) of a point located, by means of the hyperboloidal height h , out from the hyperboloidal surface, i.e. verifying the condition

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} - \frac{Z^2}{c^2} \geq 1,$$

into hyperboloidal coordinates.

Their applications range over the Geodesy field, being of interest in hyperboloidal building and cooling tower construction.

In our paper, the hyperboloidal coordinates are considered for hyperboloids of revolution (around the Z -axis). The 3D Cartesian coordinates (X, Y, Z) of a point located out from a hyperboloidal surface can be expressed in terms of a latitude ϕ , a longitude λ , and a height h measured along the hyperboloidal normal as follows:

$$X = (\nu + h) \cos \phi \cos \lambda, \tag{1}$$

$$Y = (\nu + h) \cos \phi \sin \lambda, \quad (2)$$

$$Z = (\nu(e^2 - 1) - h) \sin \phi, \quad (3)$$

where:

- ν is the prime normal section curvature:

$$\nu = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}},$$

- e is the eccentricity of the hyperboloid defined in the X - Y plane and
- a is the semi-axis of the hyperboloid defined in the X - Y plane.

They are analogously defined to the geodetic coordinates by replacing the reference ellipsoid with a reference rotational hyperboloid of one sheet. Observe that for $h = 0$, the hyperboloidal coordinates satisfy the equation of the rotational hyperboloid of one sheet

$$\frac{X^2}{a^2} + \frac{Y^2}{a^2} - \frac{Z^2}{b^2} = 1.$$

A direct transformation from Cartesian coordinates into hyperboloidal coordinates is presented, by relying on the direct transformation from Cartesian coordinates into geodetic coordinates presented in Vermeille *Direct transformation from geocentric coordinates to geodetic coordinates*. Journal of Geodesy 2002 and on the analogy between the ellipsoidal and the hyperboloidal coordinates. In addition, in the same way as in Gonzalez-Vega, Polo-Blanco. *A symbolic analysis of Vermeille and Borkowski polynomials for transforming 3D Cartesian to geodetic coordinates*. Journal of Geodesy 2009, symbolic techniques are used in order to analyze such transformation, thereby providing additional insights such as the complete characterization of the range of applicability.

It is important to mention that, from the user's point of view, our approach reduces the process of transformation from 3D Cartesian into hyperboloidal coordinates to a few mere evaluations of symbolic expressions, and is also less time consuming and more accurate than the approach presented by Feltens).

The paper would be organized as follows: the first section would present Sturm-Habicht sequences and their main properties, which will be used to completely characterize the sign behaviour of the real roots of a fourth degree polynomial P ; in the second section, the polynomial P would be generated and an algebraic method that allows the direct symbolic transformation from 3D coordinates into hc??would be provided; the third section ?? would be

devoted to present an algebraically complete classification of the plane's positive quadrant, in terms of numbers of non-zero real roots of the polynomial P ; the fourth section would present two approaches for computing the roots of the polynomial P , the first one based on the Merriman approach and the second one obtained using the Computer Algebra System **Maple**, and conjecture the algebraical equalness of the roots obtained by these approaches; in the fifth section, three examples are presented, corresponding to the three cases considered in the previous section; the last section would present several conclusions and a further research direction.

On Optimization of Skinning of Circles

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Skinning of a sequence of circles is the geometric construction of two G1 continuous curves touching each of the circles at a point, separately. A new technique has been worked out to compute skins by R. Kunkli and M. Hoffmann applying classical geometric methods and a G1 continuous skin constructed by Hermite interpolation curves. We tried to optimize the result of by segments using Slabaugh's energy minimization technique and approaching on iterative way using gradient descent procedure. These methods were not satisfied, so new techniques have been tried based on new combinations of energy functions.

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The Relation between Offset and Conchoid Constructions

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The one-sided offset surface F_d of a given surface F is, roughly speaking, obtained by shifting the tangent planes of F in direction of its oriented normal vector. The conchoid surface G_d of a given surface G is roughly speaking obtained by increasing the distance of G to a fixed reference point O by d . Whereas the offset operation is well known and implemented in most CAD-software systems, the conchoid operation is less known, although already mentioned by the ancient Greeks, and recently studied by some authors.

These two operations are algebraic and create new objects from given input objects. There is a surprisingly simple relation between the offset and the conchoid operation. As derived there exists a rational bijective quadratic map which transforms a given surface F and its offset surfaces F_d to a surface G and its conchoidal surfaces G_d , and vice versa. Geometric properties of this map are studied and illustrated at hand of some examples.

Hermite G^1 Rational Spline Motion of Degree Six

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Geometric interpolation techniques have many advantages, such as automatically chosen parametrization, lower degree of interpolants and optimal approximation order. In this talk a geometric continuous Hermite rational spline motion of degree six will be presented. The construction is divided into two parts. First, a spherical part of the motion is constructed and then the translational part is added. The nonlinear equations that determine the spherical part turn out to have a nice explicit solution. A particular emphasis will be placed on the construction of the translational part. Since the center trajectory is a G^1 continuous for an arbitrary choice of lengths of tangent vectors, additional free parameters are obtained, which affect the shape of the motion significantly. Various methods for determining these free parameters and their comparison through numerical examples, will be shown.

Bézier Nets for Spherical Powell-Sabin Interpolants

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In this talk I revisit the construction of spherical Powell-Sabin splines by Maes and Bultheel [published in CAGD 2006] and show that their spherical inversions have control nets with equal and similar properties as the Bézier nets for Powell-Sabin splines over the plane. The spherical Bézier points are not inverse to the control points of Maes and Bultheel.

Overconstrained Mechanisms Based on Special Planar Four-bars

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We start with an observation in planar Euclidean kinematics: Any circular quadrangle with vertices 1234 can be used to generate a special planar four-bar-mechanism with 1234 as vertices of the four coupler triangles - see the left hand side of the Figure. Surprisingly, this can be done in a way that for all positions of this four-bar-mechanism the four points 1234 form quadrangles similar to the initial one.

Now we extend the considerations to space and construct a 'saturated chain' of planar circular quadrangles: Any vertex of the quadrangles is connected to at least one of another quadrangle. We imbed corresponding four-bars into the planes of these quadrangles. The self-motions of the four-bar-mechanisms for general cases will guarantee self-motions of the chain with at least one motion parameter where the angle of the planes of the four-bars can be kept constant.

A physical realisation of such a mechanism can be based on the faces of a polyhedron. The vertices of the chain of circular quadrangles are placed at the edges of the polyhedron. The bodies of the planar four-bar-mechanisms can be realised by prisms with triangular bases which are linked by rotary joints. The joints between different planes (including the four-bars) have to be spherical 2R-joints.

We will work out realisations based on the faces of convex polyhedra. For these mechanisms the classical Grübler-Kutzbach-Chebyshev-formula gives a theoretical degree of freedom $F < 0$ which is why these mechanisms are supposed to be rigid in the first place. But owing to our special geometric generation physical models in general admit at least a one-parametric self-motion.

The right hand side of the Figure displays such an example based on a regular tetrahedron. The corresponding theoretical degree of freedom takes on the value $F = -14$.

C^1 Interpolation by Rational Biarcs with Rational Rotation–minimizing Directed Frames

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In this talk we present an interpolation scheme to construct rational spatial curves along with rational rotation-minimizing frames. This issue is important, for instance, when building rational motions and has applications in robotics, animation and related fields. More precisely, we consider rational parametric curves in the frame of camera planning. Then we study the construction of a smooth interpolating spatial rational curve which controls the camera motion while imaging a prescribed stationary object placed in the origin of the coordinate system. The camera orientation is given by an orthonormal directed frame, a frame for which one of its orientation vectors coincides with camera's optical axis, joining each point on the curve with the origin of the coordinate system. Since camera rotations in the image plane (the plane normal to the optical axis) are not desirable, we further require that the directed frame is rotation minimizing, i.e., the component of the rotation of the frame in the optical axis direction is identically zero.

The proposed scheme uses as input data the initial and the final curve positions and tangents on the curve, together with the associated end frame orientations. In order to keep the degree of the resulting curve and of the frame as low as possible, biarcs are used. Each curve segment is composed by two biarcs of degree 6, joining together with C^1 continuity at some unknown common point sharing some unknown tangent vector.

Four-pose Synthesis of 5R- and 6R-linkages with Rational Coupler Motion

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In our talk we present a solution to the four-pose synthesis of 5R- or 6R-linkages. These are closed kinematic chains with five or six revolute joints. Solving the synthesis problem means to compute 5R- or 6R-linkages with a one-parametric mobility whose middle link visits four given spatial poses.

So far, synthesis problems of this type have received hardly any attention. Our solution is based on a recently introduced factorization algorithm for rational motions. It transforms the mechanism synthesis problem into a rational interpolation problem on quadrics which is well-understood.

In this way, we obtain all 5R-linkages and some of the 6R-linkages that solve the synthesis problem. The extension to a Hermite-like synthesis procedure with two given poses plus instantaneous screws is straightforward. In any case, the algorithm produces several one-parametric families of linkages from which the mechanism designer can choose. We discuss some ideas for the automatic identification of feasible solutions.

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Densest Geodesic Ball Packings to $\mathbf{S}^2 \times \mathbf{R}$ Space Groups generated by Rotations

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The $\mathbf{S}^2 \times \mathbf{R}$ geometry, which is the direct product space of the spherical plane $\mathbf{S}^2$ and the real line $\mathbf{R}$, is one of the eight simply connected 3-dimensional maximal homogeneous Riemannian geometries classified by W. THURSTON. The space groups of this space have been classified by J. Z. FARKAS and E. MOLNÁR, along with the $\mathbf{S}^2 \times \mathbf{R}$ manifolds, by similarity and diffeomorphism.

In this lecture we investigate the optimal geodesic ball packings for $\mathbf{S}^2 \times \mathbf{R}$ space groups generated by rotations, compute their volumes and optimal densities of the considered packings.

E. MOLNÁR provided a unified interpretation of the Thurston geometries in the real projective 3-sphere $\mathcal{PS}^3(\mathbf{V}^4, \mathbf{V}^4, \mathbf{R})$. In our work we use this projective model to visualize the geodesic curves and spheres of $\mathbf{S}^2 \times \mathbf{R}$ on the Euclidean screen of the computer. We also show in this visualization the arrangement of the geodesic ball packings for the above space groups.

Spatial PH Spline Interpolation with Shape Constraints

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The problem of spatial interpolation with shape constraints is here addressed for the first time within the class of spatial PH spline curves. The proposed approach produces a G^1 PH spline interpolant which preserves the sign of the discrete torsion and is based on a local quintic Hermite scheme with a tension parameter for each spline segment. An asymptotic analysis in terms of the tension parameter is developed, aimed to show that in any segment it is possible to get the desired torsion sign by combining a possible reduction of such parameter with a suitable choice of the two free angular parameters characterizing any PH quintic Hermite interpolant. The numerical results show that in most cases the selection of such angles produced by the efficient CC criterion previously introduced in the literature is a good strategy. In fact it allows us to get fair PH spline interpolants, besides ensuring the satisfaction of the required shape constraints. Future research could be developed in order to define different schemes capable to ensure stronger shape constraints or higher geometric continuity to the PH spline interpolants.

Spatial Versions of Cycloid and Involute Gearing

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Study's Principle of Transference allows to transfer results from spherical geometry directly into the metric geometry of lines in the Euclidean 3-space. In practice, this is performed by extending the field of real numbers into the ring of dual numbers. Though there are some obstacles on this way, Study's principle works sometimes quite well in kinematics. Its consequences for gearing will be discussed in this presentation:

There is a unique spatial version of cycloid gearing and of the Camus Principle of Gearing. However, in a joint research with J. Angeles and G. Figliolini it turned out that the dual extension of involute gearing is not as straightforward as one might expect.

Building up a Virtual Collaboration Arena

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VirCA is a Hungarian virtual collaboration arena. It is a software framework for distributed intelligent systems. The modular framework encapsulates augmented 3D virtual reality and distributed systems. This virtual reality manager system uses communication rooms to achieve interactive collaboration. Furthermore, it has physical simulation for the interaction of the included objects. Numerous real rooms can be connected to the virtual room. In this way existing and non-existing objects can interact with each other.

The aim our work is primarily to create a virtual environment that helps people get information from complex systems and has the possibility to be extended by a three dimensional avatar figure.

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A Geometric Newton-Raphson Method for Surfaces

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We propose a generalization of the geometric Newton-Raphson method for parametric surface interrogation and intersection. Our method makes use of extrapolation along curves that interpolate local differential geometric properties. The algorithms for finding the closest point of a parametric surface to a point in space, and a point of intersection of two parametric surfaces are evaluated empirically.

Pythagorean Normals of Surfaces along Rational Curves

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A rational curve on a rational surface such that the unit normal vector field of the surface along this curve is rational will be called a curve providing Pythagorean surface normals (PSN curve). These curves represent rational paths on the surface along which the surface possesses rational offset curves. Our aim is to study rational surfaces containing enough PSN curves. The relation with PN surfaces will be investigated and revealed. The algebraic and geometric properties of PSN curves will be described using the theory of double planes.

Fat Polygons, Ball Coverings, and Collision Detection with Toleranced Motions

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One of the important tasks in robotics is the detection of collisions. There exist many algorithms for this, including time discretization, motion linearization, of bounding volumes and more. We present a novel method for the construction of simple bounding volumes or hierarchies of bounding volumes. In contrast to existing solutions, it is motion-driven. With only small computational costs, it is possible to give a no-collision guarantee for many motions of practical relevance. If a collision cannot be excluded, locus and time of possible collisions can be reported.

In our approach, the motion is a curve in 12-dimensional space of affine displacements, endowed with an object-oriented Euclidean metric. We cover this curve with balls and look at the orbits of geometric primitives (point, line, plane, polygon) with respect to these balls of affine displacements. The key property is the simple orbit shape of these primitives. It is bounded by (pieces of) spheres and hyperboloids.

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Construction and Applications of Dual Bases

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We present a novel and fast method of constructing the dual bases. As an example, we find the dual B-spline functions and show their applications in some approximation problems related to CAGD. More precisely, we use the dual B-spline functions to solve the problem of the degree reduction of B-spline curves as well as the problem of knots removal for B-spline curves.

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Generalized Hierarchical Spline Spaces

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The existing constructions for hierarchical splines are based on a sequence of tensor-product spline spaces. Thus, it is not yet possible to represent local features, such as curves with reduced order of smoothness. Moreover, these constructions are based on domains which are subsets of the plane or the space, thereby limiting the topological flexibility of the shapes that can be represented.

The talk will show how to overcome these limitations of hierarchical splines. More precisely, we will propose two generalizations of hierarchical splines. First, we will show how to generalize to more general sequences of nested underlying spline spaces, which may possibly be hierarchical spline spaces themselves. For instance, this construction can be used to model sharp features, that appear locally in the geometry considered. Second, we will propose a method to overcome the limitation of the domain to more general, manifold-type structures. By using this generalization, the hierarchical splines can be adapted to domains of general topology, while at the same time maintaining the firm mathematical basis.

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