Algebraic, Topological and Complexity Aspects of Graph Covers 2013

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Book of abstracts

Table of contents

0

Discrete group actions on orientable surfaces

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Lists of discrete group actions have many applications in different fields of mathematics. In combinatorics they can be used to derive lists of highly symmetrical maps of fixed genus: regular maps, vertex-transitive maps, Cayley maps or edge-transitive maps. The classification of actions of cyclic groups play the crucial role for enumeration problems of combinatorial objects, i.e. maps, graphs and others. Classification results can be used as an experimental material for further research, as well. The problem of classification of discrete actions of groups on orientable surfaces of genus $g \geq 2$ is considered. The classification of groups acting on the sphere is a classical part of crystallography. In case of torus the situation is known in general, though there are infinitely many group actions. Due to Riemann-Hurwitz equation (Hurwitz bound) we know that for higher genera there are just finitely many finite groups acting on a surface of a given genus. Published lists of actions go up to genus five (Broughton; Bogopolskij; Kuribayashi and Kimura). For small genera, the the classification can be done with help of computer algebra systems. Using Magma we derived the list of actions of discrete groups on surfaces of genus $2 \leq g \leq 21$ (see http://www.savbb.sk/~karabas/finacts.html).

1

MORE ON SYMMETRIC REGULAR COVERS OF TRIVALENT GRAPHS

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At last year's ATCAGC workshop (at Eugene) I presented new methods for finding symmetric regular covers of symmetric graphs, which my PhD student Jicheng Ma and I had applied to find all symmetric regular covers of K_4, $K_{3,3}$ and Q_3 with an abelian covering group. Since then we did the same for the Petersen and Heawood graphs. The case of the Heawood graph was particularly challenging, but also very interesting. For one thing, all arc-transitive groups of automorphisms of the Heawood graph act regularly on 1-arcs or on 4-arcs, yet some of the abelian regular covers are 2-arc-regular. Also the Heawood graph has 1-arc-regular covers that are obtainable in two different ways, with non-isomorphic covering groups. In this talk I will briefly summarise the new methods and the results we have found, and explain in some detail what happens with the Heawood graph.

2

Complexity of List Homomorphisms

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The list homomorphism problem generalizes several well known algorithmic problems, from the colouring and homomorphism problems to pre-colouring and surjective mapping problems. I will survey what is known for both undirected and directed graphs, with emphasis on recent results regarding the existence of polynomial or logspace algorithms.

4

TheCover

Prof. PISANSKI, Tomaz [1]

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Each finite graph on n vertices determines a special (n-1)-fold covering graph that we call TheCover. Several equivalent denitions and some surprising facts about this remarkable construction are presented. This is a joint work with Marko Boben, Aleksander Malnič and Arjana Žitnik.

5

CLASSIFICATION AND ENUMERATION OF CYCLIC REGULAR COVERINGS OF PLATONIC MAPS

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We construct and classify the regular maps which are cyclic
coverings of the platonic maps in the general case that the branched points may occur simultaneously over the vertices and face-centres of the base maps. Each such map is given by a presentation consisting of six interdependent integer parameters satisfying a system of congruence equations. The method involves studying the lattice structure of normal map-subgroups of the triangle group $\Delta(2,\infty,\infty)$ and the associated regular maps and heavily depends on the extension theory of finite groups.

6

Branched coverings of graphs and related topics

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\documentclass{article}
\usepackage{amsthm,amsfonts,amsmath,amssymb}
\usepackage[cp1251]{inputenc}
\usepackage[english,russian]{babel}
\usepackage[final]{graphicx}
\textwidth 11.5cm \textheight 16cm

\begin{document}
\begin{center}

\textbf{\Large Branched coverings of graphs and related topics}
\\
\vspace{\baselineskip}

{\large Alexander Mednykh}
\\
\vspace{\baselineskip}

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630090, Novosibirsk, Russia
\end{center}
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In this lecture we give a short survey of old and new results about branched coverings of graphs. This notion was introduced independently by many authors. See, for example, paper [1] for one of the first expositions and paper [2] for the list of references. The branched covering of graphs are also known as harmonic maps or vertically holomorphic maps of graphs. The main idea of the present talk to is create a parallel between classical results on branched covering of Riemann surfaces and those for graphs. We introduce the notion of harmonic automorphism for graph and discuss the Hurwitz upper for the number of harmonic automorphisms. Then we define a hyperelliptic graph as two fold branched covering of a tree. A few discrete versions of the Farkas and Accola theorems about hyperellipticity of coverings over genus two Riemann surface will be given.

\begin{enumerate}
\bibitem{PPJack} \emph{Parsons T.~D., Pisanski T., Jackson P.} Dual imbeddings and wrapped quasi-coverings of graphs. Discrete Mathematics. 1980. V. 31(1). P. 43--52.

\bibitem{BakerNorine} \emph{Baker B., Norine S.} Harmonic morphisms and hyperelliptic graphs. Int. Math. Res. Notes. 2009. V. 15. P. 2914--2955.

\end{enumerate}
\end{document}

7

Algorithmic aspects in the theory of graph covers

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Combinatorial treatment of graph coverings in terms of voltages has received considerable attention over the years, with its main incentive in constructing regular coverings of graphs with specific symmetry properties. Accordingly, one would like to find algorithms that would deliver answers to certain natural questions regarding symmetry issues of graphs and their coverings; this adds further motivation to the topic. In the talk I will discuss certain computational problems related to covering graph techniques.

8

Elementary abelian coverings of Platonic maps

Prof. GARETH A., Jones

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Maps are embeddings of graphs in surfaces, and regular maps are the most symmetric of these. David Surowski and I (EJC 21 (2000), 333-345, 407-418) classified the regular cyclic coverings of the Platonic maps, branched over the vertices, edges or faces. I shall describe how this can be extended to elementary abelian (and ultimately all abelian) coverings, using the G-module structure of the homology groups of a punctured sphere, where G is the automorphism group of the Platonic map. The results obtained have some overlap with those of Conder and Ma on coverings of trivalent graphs, though the methods are different.

9

Computational Complexity of Locally Constrained Homomorphisms

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We say that a homomorphism $f\colon G \to H$ is locally injective (locally surjective, locally bijective, quasi-covering, resp.), if for every vertex v of G is the mapping $f|_{N_G(v)} \colon N_G(v) \to N_H(f(v))$ injective (surjective, bijective, c_v-fold, resp.). For such mapping we define corresponding decision problem {\sc H-LIHom} ({\sc H-LSHom}, {\sc H-LBHom}, {\sc H-QCover}, resp.), where H is a fixed target graph and
the query is whether an input graph G admits a homomorphism to H of the appropriate constraint.

Full dichotomy of {\sc H-LSHom} was completed by Fiala and Paulusma in 2005.
Despite of the effort of several authors full dichotomy
of {\sc H-LIHom} and {\sc H-LBHom} have not been settled yet.
I will talk about graphs H for which the computational complexity of {\sc H-LIHom} or {\sc H-LBHom} is known. I'll also show full dichotomy for {\sc H-QCover}.

10

On the Complexity of Planar Regular Covering

Mr. KLAVIK, Pavel; FIALA, Jiri; Prof. KRATOCHVIL, Jan; NEDELA, Roman

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Let G and H be connected graphs. We say that G covers H if there exists a locally-bijective homomorphism f from G to H. For a~vertex $v \in H$, its fiber $f^{-1}(v)$ is the set of the vertices of G which are mapped by f to v. A covering f is regular if the group $\textrm{Aut}(G)$ acts transitively on every fiber of f. Roughly speaking, the copies of H in G are connected in a much more regular way which is described by some group. Therefore, regularity adds some additional algebraic structure to the covering f.

The celebrated and still unresolved Negami's Planar Cover Conjecture claims that a connected graph H has a planar cover G if and only if H can be embedded into the projective plane. Negami itself proved this conjecture in restricted settings where G is a 2-fold cover of H, and where G covers H regularly. In this talk, we present some recent results concerning the computational complexity of testing whether an input planar graph G covers regularly another input graph H.

12

On covers of doubled cycles

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In order to complete a result by C. Praeger and A. Gardiner on 4-valent symmetric graphs we apply the lifting method for elementary-abelian covering projections. In particular, for $p \noteq 2$, the graphs whose quotient over some p-elementary abelian group of automorphisms is a cycle, are described in terms of linear codes.

Joint work with A. Malni\v c and P. Poto\v cnik.